Considerations of Leakage Flux Effect on a Long Time-Constant of a Transient Eddy Current in Solid-Iron Electromagnet

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Abstract

Leakage flux of an electromagnet is considered to shorten a gap length effectively which dominates a strength of a magnetic flux density in an iron core. Because a transient eddy current occurs inside an iron core, its time constant becomes longer when the leakage flux is larger. In addition, the leakeage flux is larger when the permeability of a core is larger. A leakage flux ratio is usually several times as much as unity; so that a time constant is several times longer in comparison with that of a simple case of no leakage flux.

An analytical expression for a transint eddy current is formulated by two-dimensional simultaneous Laplace's transformation for space and time. This expression makes clear that a transient eddy current causes a spatial distribution of a flux density both at a gap and inside a core with long time constant.

1 Introduction

A transient eddy current in a core of a solid-iron electromagnet is induced by a change of an external magnetic flux density due to a sudden change of coil current with time like a step function. A spatial distribution of the transient eddy current oscillates simply in such a form as a cosine function without a skin effect occuring in the case of ac variation. The transient eddy current then disturbs the spatial distribution of the magnetic flux density with time at a gap and causes also the time-varying magnetic flux density in the core. A voltage is then induced on the coil like a reactor transformer in such a manner as non-perfect mutual coupling.

An equation for an eddy current phenomenon is same as Fourier's equation for heat conduction in principle because a displacement current in Maxwell's equations is neglected. A theoretical study on the transient eddy current phenomenon was carried out when a coil current is switched from a steady flow to zero at a given moment[1].

An experimental study on the field transition was carried out for an isochronous ring cyclotron[2]. The transitions were measured from the field strength of 1.25 T at a magnet gap to different field levels. The time constant depends on the field level and it was much shorter for the higher levels than for the lower levels. The longest was 46 minutes but the shortest was several minutes so that the time constant ranges several times depending on the final field levels.

It should be noticed that the dependence of the time constant on the field level seems to be the field derivative of the flux density of the hysteresis curve. In other words, the time constant relates to an incremental permeability at the final field level.

Because the eddy currrent occurs as time-varying phenomenon, we apply Faraday's law which includes a term of time derivative of the flux density. On the one hand, we apply Ampere's law to a magnetic field for a magnetic circuit. In order to incorporate two equations, we have to rewrite Faraday's law by a time derivative of the magnetic field as

$$\begin{aligned} rot\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ &= -\frac{\partial \mathbf{B}}{\partial \mathbf{H}}\frac{\partial \mathbf{H}}{\partial t} \end{aligned} \tag{1}$$

so that we introduce the incremental permeability

$$\mu_{\triangle} = \frac{\partial \mathbf{B}}{\partial \mathbf{H}} \tag{2}$$

in a natual manner.

In addition, in order to incorporate the incremental permeability in the time constant, we introduce a leakage flux[3] and we then assume that the leakage flux ratio depends on the permeability in an appropriate manner. Because the leakage comes from ferromagnetic characteristics of the iron core, a large permeability results in a large leakage.

2 Formalization of Eddy Current Phenomenon

2.1 Model of electromagnets with axial symmetry

A cross section of a magnet is assumed to be a uniform circle through the yoke in the shape of the closed ring with an air gap. The external field is supplied by uniform windings of a coil surrounding the ring. The symbols associating with the magnet are defined as g for the gap length, l for the yoke length, r_0 for the pole radius, and N for a turn number of coil windings. As known later, an eddy current phenomenon occurs in such a mechanism as a reactor transformer, we introduce a voltage, $V_0(t)$, which energizes a coil through a resistance of the coil, R.

We assume the field distribution with axial symmetry and it is thus adequate to choose cylindrical coordinates (r, θ, z) along the magnet axis. It is assumed from the axial symmetry that magnetic flux densities and magnetic fields have z-components only, electric flux densities and electric fields have θ-components only and a coil current and an eddy current have also θ-components only.

Because all of fields and currents have a single component, a suffix standing for a direction is omitted. On the other hand, these quantities are denoted by suffixs a and c, respectively, at the air gap and in the iron core.

2.2 Equations for transient eddy current phenomenon

When a voltage being applied to a coil is switched from a certain value at an initial steady state to another at a final steady state, a transient eddy current occurs in the iron core as a time-varying phenomenon which is a deviation from the final steady state in such a mechanism as a reactor transformer.

Ampere's law now includes an effect of an eddy current in such an expression as

$$lH_c(r,t) + gH_a(r,t) = NI(t) + \sigma l \int_r^{r_0} E(\xi,t)d\xi \quad (3)$$

where $\sigma E(r,t)$ denotes an eddy current due to an electric conductivity of the iron core, σ.

The expression is dealt with a division into two equations; the one is a radius derivative and the other is a boundary condition at $r = r_0$.

$$l\frac{\partial H_c(r,t)}{\partial r} + g\frac{\partial H_a(r,t)}{\partial r} = -\sigma l E(r,t) \quad (4)$$

$$lH_c(r_0,t) + gH_a(r_0,t) = NI(t) \quad (5)$$

Faraday's law is now meaningful due to non-zero time-varying terms as

$$\frac{1}{r}\frac{\partial}{\partial r}\{rE(r,t)\} = -\frac{\partial B_c(r,t)}{\partial t}. \quad (6)$$

In addition, Kirchhoff's law for voltage and current is incorporated as

$$RI(t) - V(t) = 2\pi r_0 N E(r_0,t). \quad (7)$$

In order to formulate the eddy current as time-varying terms, we subtract values of the final field level from these quantities and we stand for them by small letter with appropriate suffixs although electric field generating the eddy current is denoted by the same capital letter, $E(r,t)$.

For example,

$$b_c(r,t) = B_c(r,t) - B_{cf}(r), etc. \quad (8)$$

where a suffix f stands for the final field level at the steady current flow of the coil after the eddy current disappears due to its perfect decay.

Now consider the incremental permeability, $\mu_\triangle$, being kept constant and leakage flux ratio, k, being kept constant for the eddy current phenomenon

$$b_c(r,t) = \mu_\triangle h_c(r,t) \quad (9)$$

$$b_c(r,t) = kb_a(r,t). \quad (10)$$

Expressions for Ampere's law, Faraday's law, a boundary condition, and Kirchhoff's law are then rewritten as

$$\frac{\partial b_c(r,t)}{\partial r} = -\alpha^2 E(r,t) \quad (11)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\{rE(r,t)\} = -\frac{\partial b_c(r,t)}{\partial t}. \quad (12)$$

$$b_c(r_0,t) = \frac{\alpha^2}{l\sigma} Ni(t) \quad (13)$$

$$Ri(t) - v(t) = 2\pi r_0 N E(r,t) \quad (14)$$

where

$$\alpha^2 = \frac{l\sigma}{\frac{l}{\mu_\triangle} + \frac{g}{k\mu_0}}. \quad (15)$$

2.3 Two-dimensional simultaneous Laplace's transformation

In order to solve a flux density and an electric field for transient eddy current phenomenon at the same time, we perform two-dimensional simultaneous Laplace's transformation for them as

$$F(q,p) = L_{2D}\{b_c(r,t)\} \quad (16)$$

$$G(q,p) = L_{2D}\{E(r,t)\} \quad (17)$$

where q and p correspond to r and t, respectively.

After Ampere's law and Faraday's law being Laplace's transfromed and being differentiated by q appropriately, we obtain

$$F(q,p) = \frac{C(p)}{\sqrt{q^2 - \alpha^2 p}} \quad (18)$$

$$G(q,p) = -\frac{qC(p)}{\alpha^2\sqrt{q^2 - \alpha^2 p}} \quad (19)$$

where the function $C(p)$ depends p only.

Now we perform inverse Laplace's transformation about q to these equations,

$$\begin{aligned} f(r,p) &= L_q^{-1}\{F(q,p)\} \\ &= C(p)J_0(j\alpha r\sqrt{p}) \end{aligned} \quad (20)$$

$$\begin{aligned} g(r,p) &= L_q^{-1}\{G(q,p)\} \\ &= \frac{j\alpha\sqrt{p}}{\alpha^2}C(p)J_1(j\alpha r\sqrt{p}) \end{aligned} \quad (21)$$

where $J_0(x)$ and $J_1(x)$ represent 0th and 1st order Bessel's function respectively.

From the boundary condition and Kirchhoff's law being Laplace's transformed by q, we obtain a relation at $r = r_0$

$$\frac{Rl\sigma}{\alpha^2 N} f(r_0,p) - w(p) = 2\pi r_0 N g(r_0,p) \quad (22)$$

where $w(p)$ represents Laplace's transformation of $v(t)$.

Substituting the above expressions to the boundary condition, we obtain the function $C(p)$ and then obtain

$$f(r,p) = Kw(p)\frac{J_0(\frac{r}{r_0}z)}{J_0(z) - \beta z J_1(z)} \quad (23)$$

$$g(r,p) = \frac{K}{r_0\alpha^2}w(p)\frac{zJ_1(\frac{r}{r_0}z)}{J_0(z) - \beta z J_1(z)} \quad (24)$$

where K, β, and z are defined as follows;

$$K = \frac{\alpha^2 N}{Rl\sigma} = \frac{1}{\frac{l}{\mu_\triangle} + \frac{g}{k\mu_0}} \frac{N}{R} \tag{25}$$

$$\beta = \frac{2\pi N^2}{Rl\sigma} \tag{26}$$

$$z = j\alpha r_0 \sqrt{p}. \tag{27}$$

2.4 Solutions for sudden voltage change

For the simplicity, a voltage change like a step function is considered in this paper; so that $w(p)$ becomes a constant value, w_0 because $v(t)$ is treated as the delta function.

We perform the inverse Laplace's transformation about p to $f(r,p)$ and $g(r,p)$ by using Heaviside's expansion theorem. In addition, assuming a small value of β and ignoring the second term of the denominator, we obtain

$$b_c(r,t) \sim -2Kw_0 \sum_n \frac{J_0(\frac{r}{r_0} z_n)}{J_1(z_n)} e^{-\frac{t}{\tau_n}} \tag{28}$$

$$E(r,t) \sim -2\frac{K}{r_0 \alpha^2} w_0 \sum_n \frac{z_n J_1(\frac{r}{r_0} z_n)}{J_1(z_n)} e^{-\frac{t}{\tau_n}} \tag{29}$$

where z_n is the n-th zero root of 0th order Bessel's function to satisfy $J_0(z_n) = 0$ and τ_n is the time constant;

$$\tau_n = \alpha^2 \left(\frac{r_0}{z_n}\right)^2 = \frac{l\sigma}{\frac{l}{\mu_\triangle} + \frac{g}{k\mu_0}} \left(\frac{r_0}{z_n}\right)^2 \tag{30}$$

3 Discussion

3.1 Behavior of the time constant and the amplitude

The expression of the time constant can be adopted for various cases according to appropriate choice of parameters.

For a magnet with an air gap, when an incremental permeability is large enough, the time constant is obtained as

$$\tau_n \sim k\frac{l}{g}\sigma\mu_0 \left(\frac{r_0}{z_n}\right)^2. \tag{31}$$

It is known from the expression that the time constant is independent of the incremental permeability approximately.

An amplitude of the magnetic flux density at the air gap, $b_a(r,t)$, is given as

$$\frac{K}{k} \sim \frac{\mu_0 N}{gR} w_0 \tag{32}$$

because of a relation of $b_c(r,t) = kb_a(r,t)$.

It is known from the expression that the amplitude is not affected by the leakage flux ratio and the incremental permeability.

It should be noted that the incremental permeability depends on the hysteresis curve so that the time constant varies depending on the hysteresis curve.

3.2 Comparison of the prediction with the observation

We choose parameters as

$$\sigma = 5 \times 10^6 \tag{33}$$

$$\mu_0 = 4\pi \times 10^{-7} \tag{34}$$

$$\frac{l}{g} = 110 \tag{35}$$

$$r_0 = 1 \tag{36}$$

$$z_1 = 2.4 \tag{37}$$

and then we obtain the time constant as

$$\tau \sim 2k \qquad minutes. \tag{38}$$

If one assuem that a leakage flux ratio, k, varies from 3 to 24 by a factor of 8, the time constant ranges from 6 to 48 minutes.

A leakage flux ratio of 24 seems to be very large although a leakage flux ratio of 3 is acceptable. It should be noted that an incremental permeability at the low magnetic field level is very large in comparison with a value of so-called permeability.

4 Summary

We tried to formalized the transient eddy current phenomenon in the electromagnet by introducing the leakage flux. An analytical expression for a simplified model of a magnet was obtained by means of two-dimensional simultaneous Laplace's transformation for both space and time.

The time constant is expressed approximately by a leakage flux ratio, a ratio between the yoke length and the gap length, an electric conductivity of the iron core, and the permeability of the air gap. The prediction of the time constant based on the leakage flux ratio seems to be comparable with the observation which depends on the final field level when the leakage flux ratio becomes large for a large permeability.

References

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