PROPORSAL FOR A NEW TYPE WIGGLER WITH FOCUSING FOR SASE

M. Fujimoto†, R. Kato, G. Isoyama, Institute of Scientific Industrial Research, Osaka University
K. Tsuchiya, S. Yamamoto, High Energy Accelerator Research Organization

Abstract

We have studied focusing properties of a planar wiggler for FEL and SASE in the infrared region. A simple rectangular magnet model can be applied to calculate the electron trajectory in the wiggler, if the fringing field at the edge is taken into account. A new type of wiggler can be also used in SASE in the short wavelength region using a high-energy electron beam.

1 INTRODUCTION

Self-Amplified Spontaneous Emission (SASE) has attracted considerable attention as a source of high power coherent X-rays since the power of SASE grows exponentially in a wiggler up to the saturation level without an optical resonator. A wiggler of length over many tens of meters is required for such a short wavelength SASE and an electron beam with high energy (~GeV) must be transported through it. It is, however, not easy to transport the electron beam through such a long wiggler in keeping the brightness high, because a planar wiggler has no focusing in the oscillation plane. To solve this problem, focusing of the electron beam in a wiggler has been studied for SASE in the short wavelength region and some proposals for focusing schemes have been made, including focusing using the sextupole field [1].

We are studying focusing properties of a wiggler for FEL and SASE in the far infrared region. It is widely known that a planar wiggler has the focusing force in the direction perpendicular to the oscillation plane, which is proportional to the inverse square of the electron energy and to the square of the wiggler field. The focusing force of the wiggler is weak for the higher energy electron beam, while it is strong enough to focus the lower energy electron beam used for FEL and SASE in the infrared and the far infrared regions.

In this paper, we report analysis of the vertical focusing in a planar wiggler of the horizontal oscillation type. We also propose a new type of wiggler with the strong focusing force, which may be used for SASE in the shorter wavelength region.

2 THEORY

We have numerically calculated the electron orbit in the realistic magnetic field of the wiggler and evaluated the effective focusing force from the betatron wavelength in the vertical plane. Note that there is no focusing in the horizontal plane for the ideal wiggler. Fig. 1 shows the vertical betatron wavelength as a function of the oscillation amplitude y_0. The solid curve is the betatron wavelength calculated for an 11.5 MeV electron and the wiggler parameters listed in Table 1. The betatron wavelength is 0.94 m for $y_0 = 0$, which indicates that the focusing force is so strong that electrons oscillate 2 times vertically in the 1.92 m long wiggler. The wavelength is not constant and decreases as the oscillation amplitude increases due to the non-linearity of the field.

A simple model is sometimes used to estimate effects of a wiggler on the electron beam, in which the wiggler is described as a series of rectangular bending magnets in opposite directions separated by a drift space. The trajectory of the electron beam can be calculated using the transfer matrix method and the betatron wavelength thus calculated is shown by the dotted line in Fig. 1 [2]. Since the field is linear in this model, the betatron wavelength is independent of y_0. The wavelength is approximately 10 % shorter than that calculated with the realistic magnetic field for $y_0 = 0$ and therefore the focusing force is overestimated in this model. In order to reproduce the focusing force in the realistic field, we introduce the linear fringing field to the edge focusing [2]. The linear fringing field is determined such that it crosses zero at the same point as the realistic magnetic field and thus their

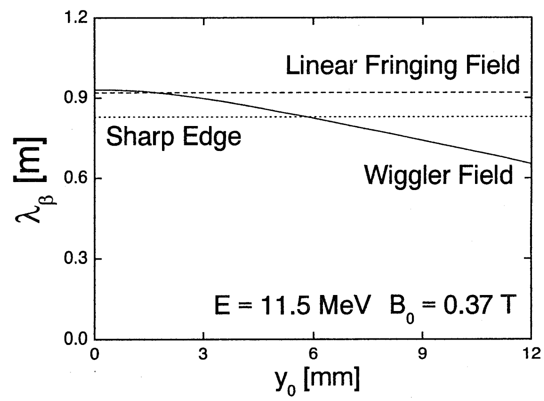

Figure 1: Batatron wavelength λ_β calculated as a function of the oscillation amplitude y_0.

Table 1: Wiggler parameters for calculations in this paper

Magnetic Period	λ_w	60 mm
No. of Period	N_w	32
Magnet Gap	g	30 mm
Block Dimensions		
Horizontal	2a	100 mm
Vertical	2b	20 mm
Longitudinal	2c	15 mm
Residual Field	B_r	1.1 T
Peak Field	B_0	0.37 T
K-value	K	1.47

†fri25@sanken.osaka-u.ac.jp

field integrals are equal. The resulting trapezoid is very close to that of the realistic field. The restoring force calculated with this model is given by

$$k = \frac{8-\pi}{3\pi}\left(\frac{e}{m_0 c}\right)^2\left(\frac{B_0}{\gamma}\right)^2, \qquad (1)$$

where e is the electron's charge, m_0 its mass, γ its kinetic energy in units of the rest mass and B_0 is the peak magnetic field of the wiggler. The betatron wavelength given by $\lambda_\beta = 2\pi/\sqrt{k}$ is calculated to be 0.93 m using parameters listed in Table 1 and shown by the broken line in Fig. 1. It agrees quite well with the wavelength calculated with the realistic model in the region around $y_0 = 0$ and the difference between them is only ~2.4 % for y_0 less than 3 mm. We will, therefore, use the linear fringing field model in the following analysis.

3 EDGE FOCUSING WIGGLER

Fig. 2 shows the schematic drawing of one period of the Edge Focusing (EF) wiggler. It is the Halbach type wiggler whose magnetic block is cut with an edge angle ϕ. In a simple model, this is described as a combination of bending magnets with the edge angle and drift spaces. An electron injected parallel to the z axis is curved to the x direction. Since the angle ε between the direction of motion and the normal of the edge is smaller than that of rectangular magnet model, the edge focusing in the vertical direction decreases and horizontal focusing is generated. The angle is given by $\varepsilon = \theta - \phi$, where θ is the maximum deflection angle of the electron given by $\sqrt{2}\,K/\gamma$. The horizontal and the vertical focusing forces calculated with the simple model are given by

$$k_h = \frac{4e}{m_0 c}\frac{B_0}{\gamma}\times\frac{\phi}{\lambda_w} \qquad (2)$$

$$k_v = k - k_h, \qquad (3)$$

where k is the natural focusing of the wiggler and given by Eq. (1). When the horizontal focusing force has a positive value and is smaller than the natural focusing force, the wiggler has focusing at the same time in the both directions (double focusing). But if the horizontal focusing $k_{h(v)}$ exceeds the natural focusing k, $k_{v(h)}$ becomes negative and the defocusing force appears in the vertical (horizontal) direction.

The magnetic field in the wiggler is numerically calculated and shown in Fig. 3,where we assume $\phi = 2°$ and the other parameters are the same listed in Table 1. The upper panel shows B_y calculated on the z axis. The letters a), b) and c) show positions where transverse variations of the magnetic field are calculated and the magnetic field at each longitudinal position is shown in the lower panel. It is seen in the lower panel that the field gradient dB_y/dx is constant in the region around x = 0 mm and it is 0.7 ~ 1.0 T/m. This field exerts a quadrupole like focusing on the electron beam.

Fig. 4 shows trajectories of 11.5 MeV electrons with different initial displacements in the y and the x directions, and they are calculated in the field presented in Fig. 3. It is apparent in the upper panel of Fig. 4 that the electrons execute betatron oscillations in the y direction around y = 0. It can be also seen in the lower panel that the electrons

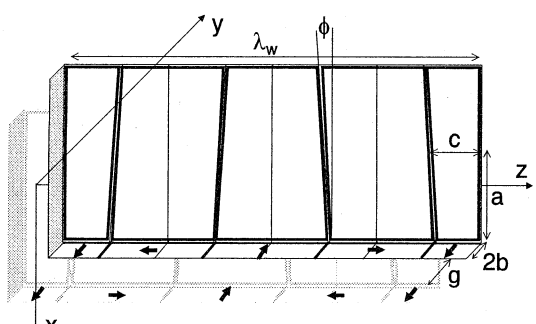

Figure 2. Schematic drawing of a single period of the Edge Focusing (EF) Wiggler.

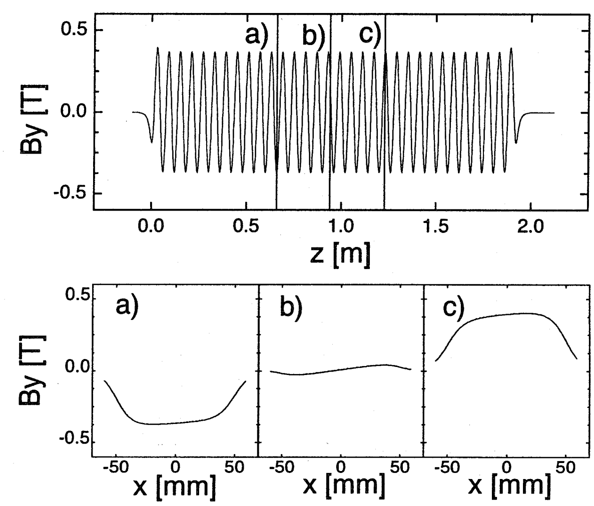

Figure 3: Magnetic field calculated with parameters listed in Table 1 and edge angle $\phi = 2°$. a), b) and c) shows the positions where transverse variation of the magnetic field in the lower panel are calculated.

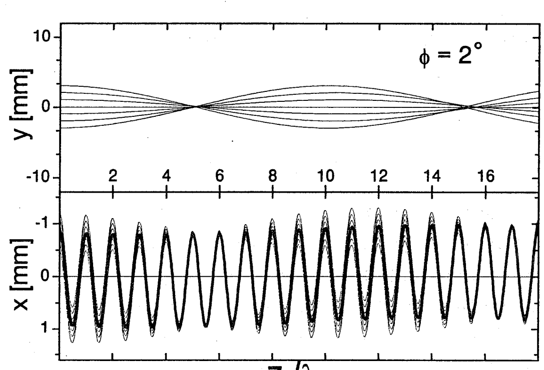

Figure 4: Trajectories of the 11.5 MeV electron calculated with the field shown in Fig. 3.

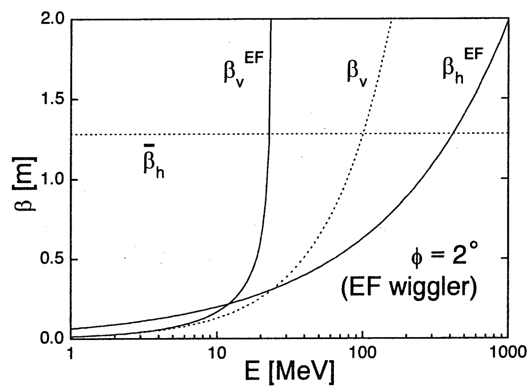

Figure 5: Betatron function as a function of electron energy. See text in details.

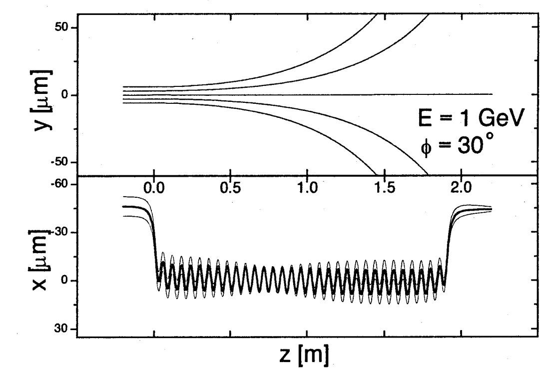

Figure 6: Trajectories of the 1 GeV electron in the EF wiggler with ϕ = 30°.

injected at points near but different from x_0 execute betatron oscillation in the x direction around the wiggling orbit of an electron injected at $x = x_0$. This indicates that the wiggler has focusing in the both directions. Betatron wavelengths obtained from the trajectories in Fig. 4 are $\lambda_{\beta x}$ = 1.33 m in the horizontal direction and $\lambda_{\beta y}$ = 1.28 m in the vertical direction, which agree quite well with betatron wavelengths $\lambda_{\beta x}$ = 1.34 m and $\lambda_{\beta y}$ = 1.28 m calculated with Eqs. (2) and (3), respectively.

4 APPLICATIONS TO SASE

The beam size can be kept at a constant value through the wiggler if there is the uniform focusing force and the electron beam is injected with the appropriate beam size [3]. This matched condition is given by $\alpha_0 = 0$ and $\beta_0 = k^{-1/2}$, where α_0 and β_0 are the Twiss parameters at the entrance of the wiggler and k is the focusing force. The beam size is given by $\sqrt{\beta\varepsilon}$, where β is the betatron function and ε is the emittance of the beam. Fig.5 shows betatron functions as a function of the electron energy calculated with the parameters listed in Table 1. The dotted curve shows the vertical matched betatron function β_v for a conventional planar wiggler with focusing force given by Eq. (1) and the dotted line shows the average value of the horizontal betatron function in the conventional wiggler, where we assume that is focused to β_h = 0.96 m at the centre of the 1.92 m long wiggler [2]. The vertical focusing force in the conventional wiggler is strong enough to make the beam size small in the lower energy region, but the focusing becomes weaker as the electron energy increases and there is practically no vertical focusing in the energy region higher than 100 MeV. The solid curves in Fig. 5 show matched betatron functions β_h^{EF} in the horizontal direction and β_v^{EF} in the vertical direction, respectively, for an EF wiggler with the edge angle ϕ = 2° which has the horizontal focusing force given by Eq. (2) and the vertical one given by Eq. (3). The natural focusing force of the EF wiggler is the same as the vertical focusing force of the conventional wiggler. The horizontal betatron function in the EF wiggler is much smaller than the average of the horizontal betatron function in the conventional wiggler, while the vertical betatron functions are almost same in the energy region lower than 10 MeV, as seen in Fig. 5. The cross sectional area of the electron beam can be made much smaller in the EF wiggler than in the conventional wiggler in the lower energy region and the gain length of SASE becomes shorter due to higher electron density. The double focusing scheme with the EF wiggler makes it possible to reduce the saturation length of SASE considerably in the longer wavelength region.

The natural focusing becomes weaker as the electron enrgy increases, as can be seen in Eq. (1). The edge focusing force, however, becomes weaker with the electron energy and besides it can be made stronger by increasing the edge angle, as can be seen in Eq. (2). In the higher energy region, the alternating focusing scheme can be used to obtain double focusing. In order to show strong focusing force with the EF wiggler in the higher energy region, we calculated trajectories of 1 GeV electrons in the EF wiggler with ϕ = 30°, where the same wiggler parameters in Table 1 are used. The upper and the lower panels show the calculated electron trajectories in the vertical and the horizontal directions, respectively. The horizontal focusing is so strong that electrons execute betatron oscillations with the wavelength $\lambda_{\beta x}$ = 3.2 m, while they are defocused in the vertical direction. The horizontal focusing and the vertical focusing can be interchanged with the negative edge angle ϕ = –30°. Combining such two types of wigglers alternately, we can make a long wiggler with double focusing for the higher energy electron beam. The alternating EF wiggler can be used for SASE in the shorter wavelength region.

5 REFERENCES

[1] G. A. Travish, PhD thesis, Univ of Califormia Los Angels (1995) p. 253 (ch. 8)

[2] H. Wiedemann, Particle Accelerator Physics (1993) (Springer-Verlag) p. 118 (ch. 5)

[3] C. A. Brau, Free-Electron Lasers (1990) (Academic Press) p. 174 (ch. 4)