EFFECT OF INJECTION-MISMATCHING UPON SLOW BEAM EXTRACTION IN AN ELECTRON STRETCHER RING

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Abstract

Beam dynamics of slow extraction in an electron pulse-stretcher ring (STB ring) at Laboratory of Nuclear Science (LNS) [1], Tohoku University has been studied by tracking simulation. Third order resonance is employed to extract circulating beam in the STB ring. Since there is no accelerating field in the ring, the beam approaches a transverse resonance condition due to synchrotron radiation loss with finite chromaticity. In this case the betatron motion of the electron is governed by the initial betatron amplitude and chromatic effect of the focusing elements in the ring. Extracted beam properties such as emittance and spill structure have been explored by a two-dimensional numerical simulation particularly for injection-mismatching in Twiss parameters.

1 PULSE-STRETCHER RING

The STB (STretcher-Booster) ring, a newly constructed pulse-stretcher ring at LNS, provides quasi-cw electron beam to study of nuclear physics and related experiments since 1996. Stored beam in the STB ring injected from a linac is slowly extracted by using the 3rd order resonance excited by a sextupole moment without RF capture. The resonating condition is created by tune shift due to energy loss via synchrotron radiation together with a finite betatron chromaticity. Since there is no sextupoles for correction of the chromaticity in the ring, a rate of the tune shift is simply determined by the natural chromaticity and the radiation loss. Accordingly acceptable energy width of the injected beam is equal to a total energy loss integrated over a repetition period of the beam injection to achieve the 100 % duty factor. Parameters related to the stretcher operation of the STB ring is shown in Table 1.

Table 1: Parameters for stretcher mode of the STB ring

Lattice type	Chasman-Green
Superperiodicity	4
Circumference	49.75 m
Beam energy	200 MeV*)
Betatron tune	(3.31, ~1.20)
Natural chromaticity	(-5.78, -4.97)
Energy loss / turn	46 eV @ 200 MeV
Repetition period (rate)	3.33 ms (300Hz)*)
Relative energy loss / period	0.46 % @ 200 MeV
Horizontal tune shift / period	0.0267 @ 200 MeV
Twiss at injection point	β_x = 20.2 m α_x = -0.06
Number of harmonic sextupole	1

*)Typical operation

2 CHOISE OF RESONATING CONDITION

2.1 Betatron amplitude at unstable fixed point and strength of sextupole moment

Defining the betatron amplitude J using Courant-Snyder invariant as

$$J = \frac{1}{2\beta}\left[x^2 + \left(\beta x' + \alpha x\right)^2\right] \quad , \tag{1}$$

then the transverse motion of an electron in a conventional phase space is described by

$$x = \sqrt{2\beta J}\cos\psi \tag{2}$$

and

$$x' = -\sqrt{2J/\beta}\left(\sin\psi + \alpha\cos\psi\right) \quad , \tag{3}$$

where β and α are the Twiss parameters, and ψ is the betatron phase. A reduced Hamiltonian including the sextupole potential is approximately written as

$$H \approx \delta J + G J^{3/2}\cos 3\phi \quad , \tag{4}$$

where δ is a deviation of fractional part of the tune from the resonance $\delta = \nu - l/3$, and G is a Fourier amplitude of the sextupole moment averaging over the ring circumference, which is calculated for a driving term $3\nu = l$ as [2],

$$G e^{i\xi} = \frac{1}{2\pi}\oint \frac{\sqrt{2}}{12}\beta^{3/2}S(s)\exp i\left[3\int_0^s \frac{ds}{\beta} - (3\nu - l)\frac{s}{R}\right]ds , \tag{5}$$

where $S(s)$ and R are the sextupole field strength normalized by the beam rigidity and the mean radius of the ring, respectively. A new phase ϕ is expressed by $3\phi = 3\psi + l\theta - \xi$, and the orbiting angle θ is s/R.

From the Hamiltonian eq. (4), a betatron amplitude at an unstable fixed point (*ufp*) of $\phi = \pi$ is found to be

$$J_{ufp} = \frac{4}{9}\left(\frac{\delta}{G}\right)^2 \quad . \tag{6}$$

Because only one harmonic sextupole is located at the center of a long straight section in the STB ring where α

is 0, and an extraction wire septum is at just downstream of it, the ustable fixed point in the real space is approximately derived from

$$x_{ufp} = -\sqrt{2\beta}\,\frac{2\delta}{3G} \tag{7}$$

It is obvious that the resonating condition is defined by the tune deviation δ and the Fourier strength G. In the STB ring the wire septum is located on 3 cm from the central orbit, therefore x_{ufp} should be of course less than that.

2.2 3-turn separation

The most significant feature for optimization of the resonating condition is an increase of horizontal displacement at each successive 3-turn. It may results from a compromise between the emittance of extracted beam and the extraction efficiency due to finite thickness of the septum wire. Using Hamiltonian s equations motion derived from eq. (3), a double differential equation for the betatron amplitude can be obtained as

$$\frac{\partial^2 Z}{\partial \theta^2} = 3\,\delta^2 Z + 6\,\delta\, Z^2 \quad Z \equiv J\,/\,J_{ufp} - 1 \tag{8}$$

In order to find an appropriate sextupole strength, a numerical calculation using eq. (8) was done for various δ and J_{ufp}. Figure 1 show the relative 3-turn separation as a function of the relative horizontal displacement of the particle in the real space.

For instance, if we chose the location of *ufp* to be 1/2 of the septum position and δ = - 0.004 for the resonating tune in the STB ring, the 3-turn separation would be 3 mm. To obatin a small emittance for the extracted beam, the 3-turn separation should be small, but sufficiently larger than the wire thickness to secure a better extraction efficiency. Consequently we have chosen an operation point of x/x_{ufp} = 1.5 and δ = - 0.003. In this case the sextupole strength is - 0.377 m^{-2}, and the 3-turn separation is expected to be ~ 1.2 mm at the wire whose thickness is 0.3 mm.

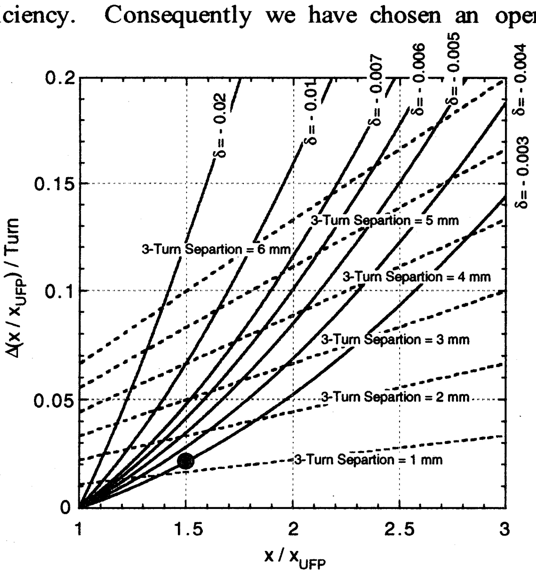


Figure 1: Calculated turn separation of resonated electron for various resonating condition. A closed circle indicates an operating point chosen in the simulation.

3 TRACKING SIMULATION

3.1 Matched injection with Δp/p = 0

Single particle equations of motion are derived from a Floquet transformed Hamiltonian including the sextupole potential V_3

$$H = \nu_x J_x + \nu_y J_y + V_3(s) \tag{9}$$

$$V_3(s) = -\frac{\sqrt{2}}{4} J_x^{1/2} J_y \beta_x^{1/2} \beta_y S(s)\left(2\cos\phi_x + 2\cos\phi_x \cos 2\phi_y\right) + \frac{\sqrt{2}}{12} J_x^{3/2} \beta_x^{3/2} S(s)\left(\cos 3\phi_x + 3\cos\phi_x\right) \tag{10}$$

where

$$\phi_{x,y} = \psi_{x,y} + \int_0^s \frac{ds}{\beta_{x,y}} - \nu_{x,y}\,\theta \tag{11}$$

In the simulation, the emittance of the injected beam is assumed to be 300 nmrad for both x and y planes, which is not far from measured values, and chromatic tune shifts due to synchrotron radiation loss are added at each turn. Initial betatron amplitude is controlled by a height of the bump orbit so as to that the center of the electron distribution in the horizontal phase space is on the separatrix.

To simplify the particle dynamics in the stretcher ring, a beam at a certain energy without the energy spread is investigated by the simulation. Since the resonating tune is 3.3303, the initial tune of the injected beam is employed to be 3.3164 to which the beam is resonated in the middle of the repetition period (see related parameters in Table 1).

Figure 2 shows a time structure of spilled-out electrons. Because of finite beam emittance and then finite initial betatron amplitude, each particle resonates at respective tune deviation δ. Accordingly the spill structure is reflected by distribution of the phase space

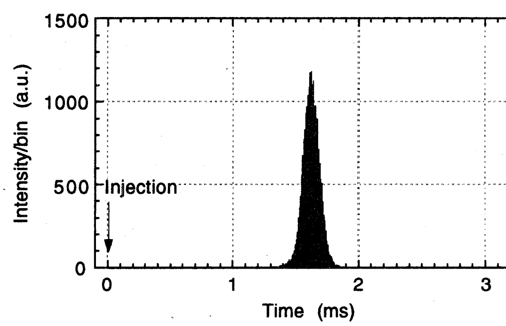


Figure 2: Time distribution of spilled-out electrons. Gaussian distribution of the phase space density for the injected beam is employed in the simulation.

density of the injected beam. That is also an origin of spreading of the emittance of the extracted beam. In Fig. 3, Poincare plot of the spilled-out beam is shown together with that of circulating beam at 0.8 ms after the injection. Filamentation due to the spread of the betatron amplitude is clearly seen.

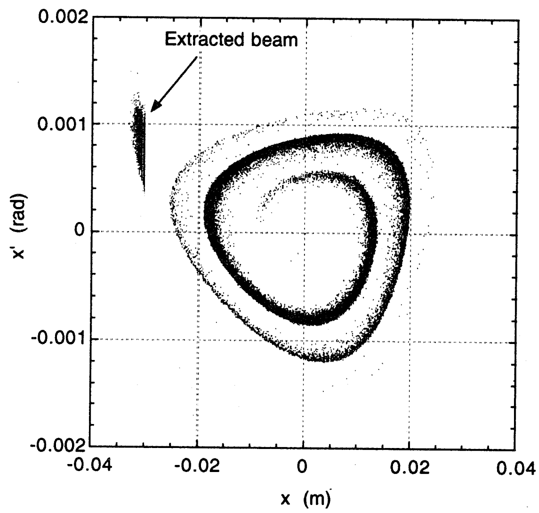
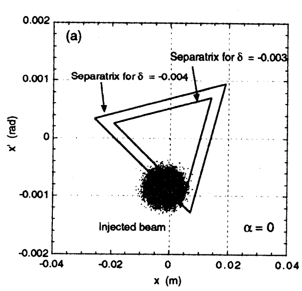


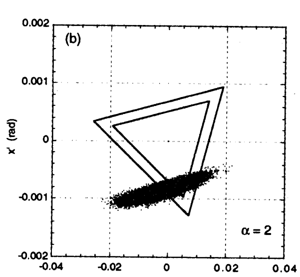
Figure 3: Phase space distributions of the extracted beam and the circulating beam before resonating at the location of the wire septum.

3.2 *Mismatched injection with $\Delta p/p = 0$*

Twiss parameters of the injected beam is adjusted to be identical to the lattice function at the injection point in the STB ring so far, i.e., $\beta = 20$ m, $\alpha = 0$, which is a fundamental strategy for high efficient beam injection. However this is not always valid for the stretcher ring because the instantenaous betatron tune of the injected beam is already close to the resonating tune. It may become clear by seeing simulation results for mismatched injections.

Two typical mismatched injections are examined as





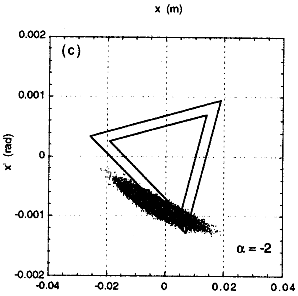


Figure 4: Initial phase space at the extraction wire septum for the first turn: matched injection α=0 (a), mismatched injections α= 2 (b) and -2 (c). Lines show separatrices for δ = -0.003 and -0.004 with a same sextupole strength.

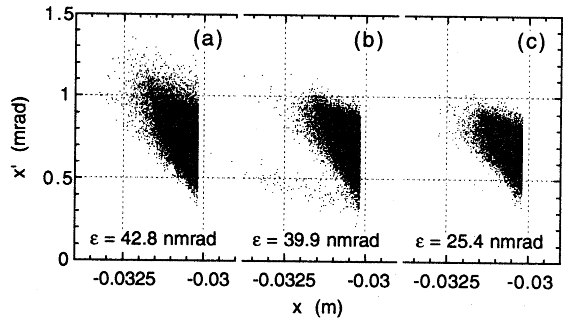


Figure 5: Phase space distributions of the extracted beams. Extraction efficiencies are 72, 68 and 71 % for the cases of (a), (b) and (c), respectively.

shown in Fig. 4. Phase space distributions of the extracted beams are shown in Fig. 5, and deduced emittances are also denoted.

It is obvious that a mismatched injection of the case (c) provides a smaller emittance beam. Although the beam of (b) is apparently better than (a), one can see that a part of injected beam gets into out of the separatrix immediately after the injection, and less extraction efficiency. It can be said that the mismatched injection of (c) is rather matched for the resonating phase space, which is seen in Fig. 4 (c).

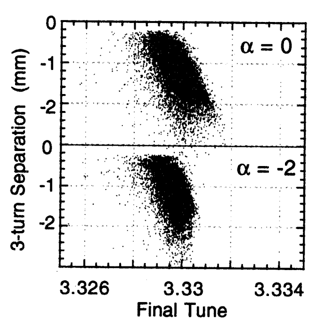


Figure 6: 3-turn separation at the wire septum plottedd as a function of the final resonating tune, where maximum acceptable energy width for the extraction ($\pm$ 0.23 %) is employed for the initial condition of the injected beam.

4 CONCLUSION

Importance of phase space matching with the separatrix is also seen in the 3-turn separation and the final tune distribution shown in Fig. 6.

Because the STB ring is a pulse stretcher without RF capture and thus no damping effect, the beam injection scheme is crucial. Size and phase of the separatrix at the injection point is derived from Fourier transform of distributed sextupole fields in the ring. In order to optimize the injection matching for production of high quality extraction beam, proper location of the sextupole magnets has to be considered.

REFERENCES

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[2] S.Y. Lee, Accelerator Physics, World Scientific Publishing Co., River Edge, NJ, USA (1999).