Experimental measurement of resonance islands at the Photon Factory storage ring

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Abstract

The coherent betatron motions after a horizontal kick were examined with various field strengths of the octupole magnets near the third-order resonance line($3\nu_x = 29$), at the KEK-Photon Factory electron storage ring (PF-ring). At a particular condition, the particles were trapped in the resonance islands. The responses depended on the strength of octupole magnetic fields. As a result, we found that the motion of the particles near the resonance were classified into three regions for the strength of the octupole field, and the transition from stable trapped region to damped region was caused by a slight difference of the octupole field.

1 INTRODUCTION

The nonlinear beam dynamics of the transverse betatron motion in circular accelerator has been studied using analytical, numerical and experimental methods [1, 2]. Despite the progress in explanation of nonlinear phenomena, there still is a gap between analytical, numerical prediction and reality. Especially, perturbation theory is limited near a resonance. To reduce this gap, experimental nonlinear-beam-dynamics studies have become increasingly important. We have been studied experimentally the nonlinear-beam-dynamics near the vertical third-order resonance [3].

In the PF-ring, to suppress transverse instabilities, four octupole magnets are installed. The octupole magnets can produce the large amplitude dependent tune shift to beam. This effect near a resonance has attracted considerable interest. This paper concerns the experimental study of phase space topology under various octupole field strengths near the horizontal third-order resonance.

2 EXPERIMENTS

The principal parameters of the PF-ring under the low-emittance optics are given in Table 1.

In order to study the phase space topology near the horizontal third-order resonance, transverse phase space monitor system was used [4]. The phase space monitor system consists of fast kicker magnets and turn-by-turn monitors. The fast kicker magnets provide beam with a large coherent motion. The coherent betatron motion was measured until 16383 turn by using the turn-by-turn monitors. To measure the coherent betatron motion in the phase space, two BPMs at a long straight section are used. The beam position x and the beam angle x' are obtained from the data of two positions.

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Table 1: Principal parameters of the Photon Factory storage ring under the present low-emittance optics.

Parameter	Symbol	Value
Beam energy	E	2.5 GeV
Circumference	C	187 m
Harmonic number	h	312
Horizontal betatron tune	ν_x	9.60
Vertical betatron tune	ν_y	4.28
rf frequency	f_{rf}	500.1 MHz
Revolution period	τ	624 nsec
Emittance	ϵ_x, ϵ_y	36, 0.36 nmrad
Energy spread	σ_ϵ	0.00073
Beam size	σ_x, σ_y	0.58, 0.04 mm

The experiment was performed in the single-bunch operation mode. The initial stored current was set to be about 5 mA. To measure the horizontal coherent betatron motions near the third-order resonance line($3\nu_x = 29$), the initial betatron tunes were selected near $(\nu_x, \nu_y) = (9.66, 4.28)$.

In this experiment the octupole current I_o was changed from -20.0 to 10.0 A. Then, the horizontal tune shift due to the octupole magnets is estimated to $\Delta\nu_x \sim -30.8 \cdot J_x I_o$, where J_x is a following action variable.

3 DATA ANALYSIS

For particle motion in circular accelerator, the deviation from the closed orbit, $x(s)$, satisfies Hill's equation:

$$\frac{d^2x}{ds^2} + K(s)x = \frac{\Delta B_y}{B\rho}. \quad (1)$$

Where $K(s)$ is a function of the quadrupole strength, $B\rho$ is the magnet rigidity, and s is the longitudinal particle coordinate. When the betatron motion is linear, Hill's equation can be solved using the Floquet transformation [5] to obtain the solution

$$x(s) = \sqrt{2\beta_x J_x}\cos\phi, \quad (2)$$

where J_x and ϕ are action-angle variables, β_x is the horizontal betatron amplitude function. Normalized momentum is given by

$$p_x = \alpha_x x + \beta_x x', \quad (3)$$

where $\alpha_x = -1/2 \cdot d\beta_x/dx$. When linear motion is plotted in x-p_x space, it is a circle defined by the equation,

$$p_x^2 + x^2 = 2\beta_x J_x. \quad (4)$$

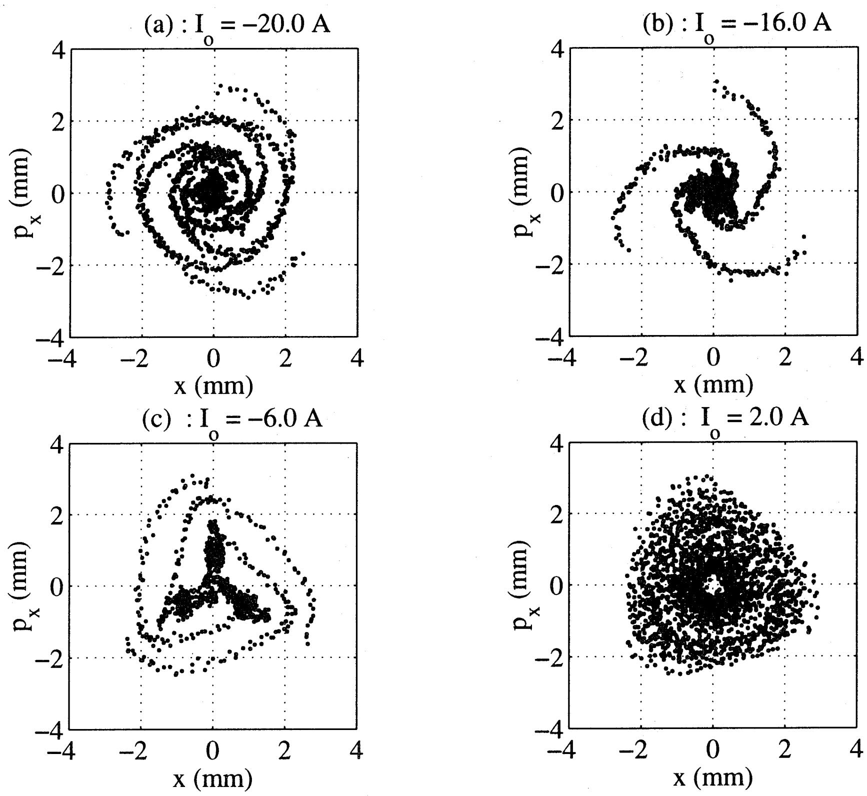

Figure 1: The Poincarè map (x-p_x) near the horizontal third-order resonance.

The values of α_x and β_x, needed to transform the position variables to the normalized momentum, were determined by fitting experimental data to a circle, because the equation (4) becomes the equation of circle when the motion is linear. Using this method the Courant-Snyder parameters were determined to be $\alpha_x = 0.092$, $\beta_x = 4.99$ m by experimental data with small amplitude.

4 RESULT

In order to search a condition when the particles are trapped in the resonance islands, we changed the initial betatron tunes and the horizontal amplitude. Under many experimental conditions, the particles were not trapped in the islands, and the coherent motions were damped. However, at a particular condition, the particles were trapped in the islands. This condition was very sensitive to the initial betatron tunes and the horizontal amplitude.

Under this condition, we examined the motion of the particles near the resonance with various octupole excitation currents (from -20A to 10A, -1A step). Fig.1 displays a horizontal phase space (x-p_x) plot (Poincarè map) for four different octupole currents. In Fig.1(a), (b) and (c), the motion was influenced by the resonance, the distribution in the phase space was localized. Especially, in Fig.1(c), the particles were tapped in the islands stably. In Fig.1(d), the motion was damped, the distribution in the phase space was not localized.

When the particles were trapped in the islands stably, the horizontal position x was periodic in every three turn, and it was divided into three positions, as shown in Fig.2. On the other hand, when the particles were not trapped, the position x was not periodic.

5 DISCUSSION

In order to estimate the influence of the third-order resonance, we calculate the cumulant for the angle ϕ. When the particles are trapped in the islands, the angle ϕ_j is peri-

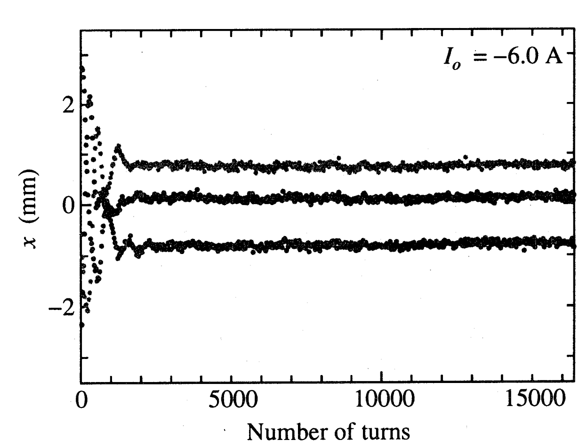

Figure 2: The coherent betatron motion at a octupole excitation of -6.0 A. The particles were trapped in the islands.

odic in every three turn as shown in Fig.3, where j is turn number (j = 1, 2, ..., 16383). Then the angle ϕ_j can be divided into $\phi_{1,i} = \phi_{3i-2}$, $\phi_{2,i} = \phi_{3i-1}$ and $\phi_{3,i} = \phi_{3i}$, where i = 1, 2, ..., 5461(=N). If the particles are trapped in the islands, the $\phi_{n,i}$ converges on a certain value. On the other hand, if the particles are not trapped, the distribution of $\phi_{n,i}$ is uniform from $-\pi$ to π. Here, we defined the cumulant of the angle $\phi_{n,i}$:

$$\langle(\phi - \langle\phi\rangle)^2\rangle = \frac{1}{3N}\sum_{n=1}^{3}\sum_{i=1}^{N}(\phi_{n,i} - \langle\phi_n\rangle)^2, \quad (5)$$

where the average of $\phi_{n,i}$ is

$$\langle\phi_n\rangle = \frac{1}{N}\sum_{i=1}^{N}\phi_{n,i}. \quad (6)$$

When ϕ_j has a uniform distribution, the cumulant is calculated to be $\pi^2/3$. However, when the particles are influenced by the resonance, the cumulant approaches zero.

The cumulants for experimental data are shown in Fig.4. Then, we find that the behaviors of the motion are classified into following three regions for the strength of the octupole current using this cumulant. In region 1 (-20 $\leq I_o \leq$ -11A), the particles are trapped in the islands, but the motion is unstable, then the cumulant increases with decrease of the octupole current. In region 2 (-10 $\leq I_o \leq$ -4A), the cumulant approaches zero as the particles are trapped stably. In region 3 (-3 $\leq I_o \leq$ 10A), the motion is damped, and it is weakly influenced by the resonance, then the cumulant converges on about $\pi^2/3$. Therefore, the influence of the third-order resonance can be characterized clearly using this cumulant.

As shown in Fig.4, the transition from region 2 to region 3 is caused by a slight difference of the octupole current. Then, the difference of the tune shift due to the octupole magnets is estimated to be very small ($\Delta\nu_x \sim -3 \times 10^{-5}$). We guess that these behaviors are caused by a slight difference of the tune shift.

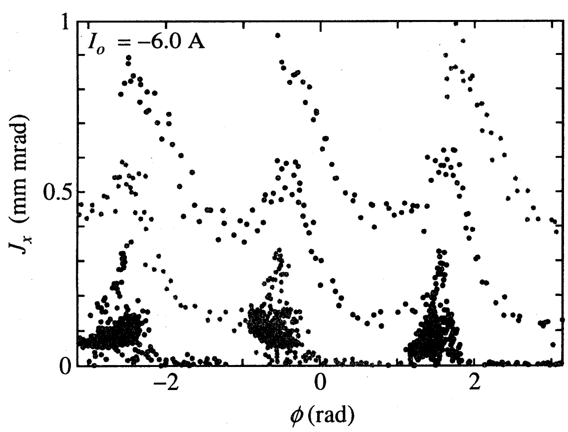

Figure 3: The Poincarè map in action-angle variables for the particles trapped in third-order resonance islands at a octupole excitation current of -6.0 A.

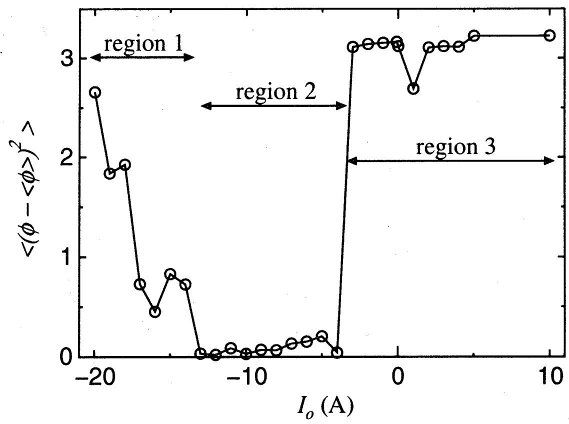

Figure 4: Cumulant of angle $\langle(\phi - \langle\phi\rangle)^2\rangle$. The influence of the third-order resonance can be characterized using this cumulant.

6 SUMMARY

In this experiment, the coherent motion after a horizontal kick was examined with various field strengths of the octupole magnets near the third-order resonance line. And, we calculate the cumulant for the angle ϕ, in order to estimate the influence of the third-order resonance. Using this cumulant, the influence of the resonance can be characterized clearly. We guess that the behavior of the motion in the resonance islands is caused by a slight difference of the amplitude dependent tune shift due to the octupole magnets. Now we are going to understand the phenomena through theoretical and numerical approach in detail.

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