Analysis of an FEL oscillator at zero detuning length of an optical cavity in the early stage of the field evolution

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Abstract

The evolution of the field of an FEL oscillator at zero detuning length of an optical cavity (δL = 0) is studied analytically. The field on the leading edge at the first round trip is the same as that of a self-amplified spontaneous emission (SASE) FEL characterized by an FEL parameter ρ. The field evolves with round trips by interaction with electrons. The field in the early stage of the evolution is found to scale with the FEL parameter ρ and the round-trip number, and is similar to that of SASE with high electron beam density.

INTRODUCTION

An experiment in the Japan Atomic Energy Research Institute (JAERI) has shown that an efficiency of an FEL oscillator can become maximum at the zero detuning length of an optical cavity ($\delta L = 0$) despite the lethargy effect [1]. The sharp increase of an efficiency near zero detuning length has been also observed in BINP or KAERI, recently [2]. A time dependent simulation including shot-noise effects has successfully reproduced the efficiency detuning curve obtained in the experiment in JAERI-FEL [3], but the physics responsible for the FEL at $\delta L = 0$ has not been clearly proposed yet. A few theoretical studies have followed the experiment and attributed the sideband instability [4] or the superradiance in short-pulse FELs [5] to the lasing at $\delta L = 0$. However, those studies are still based on numerical simulations.

The main difference of FELs between $\delta L = 0$ and $\delta L < 0$ is whether incident electrons interact with the field characterized by the steep intensity gradient on the leading edge similar to SASE [6]. At $\delta L < 0$, an optical field is pushed forward with round trips, and electrons interact with a field much stronger than that on the leading edge at $\delta L = 0$. An analysis of the interaction between the field similar to SASE and electrons has been performed recently [7]. The analysis shows that intense few-cycle FELs are generated as a result of intensive energy transfer from electrons to the field at the peak at $\delta L = 0$. The present study focuses on the early stage of the field evolution at $\delta L = 0$, and shows that the field scales with the FEL parameter ρ [8] and the round-trip number n during the evolution.

BASIC EQUATIONS

The present study is performed under the slowly varying envelope approximation (SVEA) [9]. The initial electron energy $\gamma_0 mc^2$ is assumed to be resonant for radiation wavelength $\lambda = \lambda_w(1 + a_w^2)/(2\gamma_0^2)$, where $\lambda_w = 2\pi/k_w$ and a_w are undulator period and parameter respectively. In order to deal with few-cycle fields, I choose unity for the number of undulator periods, through which electrons pass in a scaled time, instead of $1/(4\pi\rho)$ [8] or the total number of undulator periods N_w [10]. The fundamental FEL parameter ρ [8] is defined by

$$\rho = [ea_w F\sqrt{n_e/(\epsilon_0 m)}/(4ck_w)]^{2/3}/\gamma_0 \quad (1)$$

in MKSA units, where n_e is the electron beam density and F is unity for a helical undulator or Bessel function $[JJ]$ for that of planar type [8]. I define dimensionless time by $\tau = ct/\lambda_w$, dimensionless optical field by

$$a(\zeta,\tau) = \frac{2\pi e a_w \lambda_w F}{\gamma_0{}^2 mc^2} E(\zeta,\tau)\exp[i\phi(\zeta,\tau)] \quad (2)$$

with phase $\phi(\zeta,\tau)$, and dimensionless beam current by $[4\pi\rho(\zeta,\tau)]^3$ as similar to Ref. [10]. Here $E(\zeta,\tau)$ is the rms optical field strength. The longitudinal position, dimensionless energy and phase of the i-th electron are respectively defined by $\zeta_i(\tau) = [z_i(t) - ct]/\lambda$, $\mu_i(\tau) = 4\pi[\gamma_i(t) - \gamma_0]/\gamma_0$ and $\psi_i(\tau) = (k_w + k)z_i(t) - \omega t$. In the present definition, the electron dynamics is represented by the following pendulum equations [10]:

$$\frac{d\mu_i(\tau)}{d\tau} = 2|a[\zeta_i(\tau),\tau]|\cos\{\psi_i(\tau) + \phi[\zeta_i(\tau),\tau]\}, \quad (3)$$

$$d\psi_i(\tau)/d\tau = \mu_i(\tau). \quad (4)$$

The evolutions of FEL phase and amplitude are given by [10]

$$\partial\phi(\zeta,\tau)/\partial\tau = [4\pi\rho(\zeta,\tau)]^3/|a(\zeta,\tau)|\times \langle\sin\{\psi_i(\tau) + \phi[\zeta_i(\tau),\tau]\}\rangle_{\zeta_i=\zeta}, \quad (5)$$

$$|a(\zeta,\tau)|/\partial\tau = -[4\pi\rho(\zeta,\tau)]^3\times \langle\cos\{\psi_i(\tau) + \phi[\zeta_i(\tau),\tau]\}\rangle_{\zeta_i=\zeta}. \quad (6)$$

The angular bracket shows an average over electrons around ζ within λ along the propagating direction [9].

FIELD AT THE 1ST ROUND TRIP

A field with uniform phase over length of $N\lambda$ is formed from an initial incoherent field after passage of electrons through N undulator periods [11]. This formation corresponds to the spectrum narrowing in the frequency domain [6, 12], and directly leads to a backward coherent state. The average of the coherent field at time τ is approximately given by the solution of the cubic equation

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for an input weak field with uniform amplitude and phase $a(0) = |a(0)|e^{i\phi(0)}$ [10]:

$$a(\zeta,\tau) = [a(0)/3][\exp(4\pi\rho\tau e^{i\frac{\pi}{6}}) + \exp(4\pi\rho\tau e^{i\frac{5\pi}{6}}) + \exp(4\pi\rho\tau e^{-i\frac{\pi}{2}})]. \quad (7)$$

The coherent field is ranged at $\zeta < -\tau$, and corresponds to the steady state regime described in Ref. [8]. Since the evolution of the field at $\zeta = -\tau$ stops at time τ due to the slippage of electrons, the field at $\zeta \geq -\tau$ is given by

$$a(\zeta) = [a(0)/3][\exp(-4\pi\rho\zeta e^{i\frac{\pi}{6}}) + \exp(-4\pi\rho\zeta e^{i\frac{5\pi}{6}}) + \exp(-4\pi\rho\zeta e^{-i\frac{\pi}{2}})]. \quad (8)$$

This field corresponds to the leading edge described in Ref. [8]. In the high gain regime defined by $-\rho\zeta > 0.1$ or $2(-4\pi\rho\zeta)^3 > 4$, the field on the leading edge is approximately given by

$$a(\zeta) = [a(0)/3][\exp(-4\pi\rho\zeta e^{i\frac{\pi}{6}}). \quad (9)$$

The phase of $a(\zeta)$ is given by

$$\phi(\zeta) = \phi(0) - 2\pi\rho\zeta. \quad (10)$$

FIELD AT THE NTH ROUND TRIP IN THE EARLY STAGE OF THE EVOLUTION

The output field at the first round trip in an oscillator is identical to that of SASE given by Eq. (9), and becomes an input field for the 2nd round trip. The head of a round-trip FEL coincides with that of an incident electron pulse at the entrance to an undulator at $\delta L = 0$. Therefore a study of the interaction with the field given by Eq. (9) and electrons is indispensable to understand the FEL evolution at $\delta L = 0$.

Figure 1 shows a semi-log plot of an FEL amplitude at saturation as a function of longitudinal position in units of resonant wavelength λ. This is obtained in a time dependent simulation. The right hand side shows the front edge of the FEL field. The zero is the position of the head of incident electrons at the entrance to an undulator. The position of the principal peak of the field is represented as ζ_p. The region from 0 to ζ_p is the leading edge. The inset shows the linear plot of the amplitude.

I assume that the output field on the leading edge at the round-trip number n is the same as that of SASE with FEL parameter ρ_n. The assumption is exact at the 1st round trip of an oscillator where $\rho_1 = \rho$. The validity of the assumption for the subsequent round trip will be confirmed later. Under the assumption, the field on the leading edge is given by

$$a_n(\zeta) = [a_n(0)/3]\exp[-4\pi\rho_n\zeta e^{i\pi/6}]. \quad (11)$$

The phase of the ith electron remains almost unchanged during the interaction with the field given by Eq. (11): $\psi_i(\tau) \approx \psi_i(0)$. This is because the field on the leading edge is weak as shown in Fig. 1 and the synchrotron

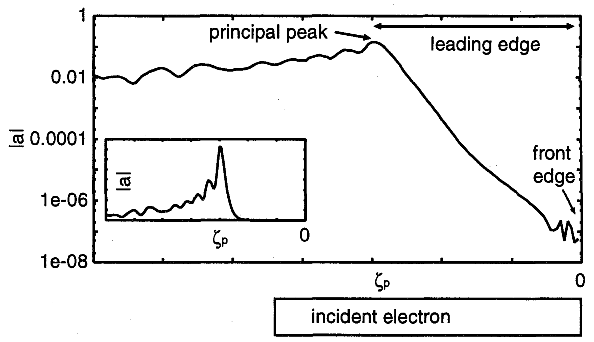

Figure 1: Semi-log plot of an FEL amplitude $|a|$ at $\delta L = 0$ after saturation with respect to the longitudinal position ζ (solid line) together with an electron pulse at the entrance to an undulator. The positions of the front edge and the principal peak are 0 and ζ_p, respectively. The inset is a linear plot of $|a|$.

oscillation of electrons occur at ζ_p. The energy modulation given to the ith electron at τ' is represented by $\Delta\mu_i(\tau') = 2|a_n[\zeta_i(\tau')]|\cos\{\psi_i(0) + \phi_n[\zeta_i(\tau')]\}\Delta\tau'$ from Eq. (3). The phase modulation at τ due to $\Delta\mu_i(\tau')$ is given by $\Delta\mu_i(\tau')(\tau - \tau')$ from Eq. (4). The phase of the ith electron is derived from the sum of those phase modulations during τ:

$$\psi_i(\tau) = \psi_i(0) + 2|a_n[\zeta_i(0)]|\int_0^\tau e^{2\pi\sqrt{3}\rho\tau'} \times \cos\{\psi_i(0) + \phi_n[\zeta_i(\tau')]\}(\tau - \tau')d\tau'. (12)$$

Equation (12) can be used when the gain of the field is low and the field remains almost unchanged during pass through an undulator. Substitution of Eq. (11) into Eq. (12) yields

$$\psi_i(\tau) = \psi_i(0) + \{|a_n[\zeta_i(\tau)]|/(8\pi^2\rho^2)\}\times \Big(\cos\{\psi_i(0) + \phi_n[\zeta_i(\tau)] - \pi/3\} - e^{-2\pi\sqrt{3}\rho\tau}\cos\{\psi_i(0) + \phi_n[\zeta_i(0)] - \pi/3\} - 4\pi\rho\tau e^{-2\pi\sqrt{3}\rho\tau}\cos\{\psi_i(0) + \phi_n[\zeta_i(0)] - \pi/6\}\Big). \quad (13)$$

The phase shift and gain of $a(\zeta,\tau)$ due to an electron micro-bunch in units of λ, whose initial positions are around $\zeta + \tau$, are obtained by substituting Eq. (13) into Eqs. (5) and (6) respectively as follows:

$$\partial\phi_n(\zeta,\tau)/\partial\tau = 4\pi(\rho^3/\rho_n^2) \times \{1/2 - e^{-2\pi\sqrt{3}\rho_n\tau}[\cos(2\pi\rho_n\tau + \pi/3) + 4\pi\rho_n\tau\cos(2\pi\rho_n\tau + \pi/6)]\}, \quad (14)$$

$$[\partial|a_n(\zeta,\tau)|/\partial\tau]/|a_n(\zeta,\tau)| = 4\pi(\rho^3/\rho_n^2) \times \{\sqrt{3}/2 - e^{-2\pi\sqrt{3}\rho_n\tau}[\sin(2\pi\rho_n\tau + \pi/3) + 4\pi\rho_n\tau\sin(2\pi\rho_n\tau + \pi/6)]\}, \quad (15)$$

when $|a_n(\zeta)|/(8\pi^2\rho_n{}^2) \ll 1$ is satisfied or synchrotron oscillation does not occur.

The field gain per round trip due to electrons $[\partial a_n(\zeta)/\partial n]/a_n(\zeta) = [\partial|a_n(\zeta)|/\partial n]/|a_n(\zeta)| + i[\partial\phi_n(\zeta)/\partial n]$ is derived from integrations of Eqs. (14) and (15) from 0 to $-\zeta$ as follows:

$$[\partial a_n(\zeta)/\partial n]/a_n(\zeta) = (\rho/\rho_n)^3[-4\pi\rho_n\zeta e^{i\pi/6}\times (1+e^{4\pi\rho_n\zeta e^{i\pi/6}}) - 2(1-e^{4\pi\rho_n\zeta e^{i\pi/6}})]. \quad (16)$$

Equation (16) is asymptotically equal to

$$[\partial a_n(\zeta)/\partial n]/a_n(\zeta) = (\rho/\rho_n)^3(-4\pi\rho_n\zeta e^{i\pi/6} - 2), \quad (17)$$

for $\rho_n\zeta < -0.2$. The differentiation of Eq. (11) by n gives the field evolution per round trip:

$$[\partial a_n(\zeta)/\partial n]/a_n(\zeta) = -4\pi e^{i\pi/6}(\partial\rho_n/\partial n)\zeta. \quad (18)$$

When the gain from electrons represented by Eq. (17) is much higher than $\alpha/2$, the gain is equal to the field evolution given by Eq. (18). This leads to the evolution of ρ_n as a function of n:

$$\rho_n \approx \rho(3n-2)^{1/3}. \quad (19)$$

Equation (19) confirms the assumption that the field on the leading edge at the round-trip number n is the same as that of SASE with FEL parameter ρ_n. As the slope of the amplitude increases with ρ_n, the peak intensity on the leading edge increases and the pulse length becomes short.

COMPARISON WITH A CALCULATION

The gain of the field at the 2nd round trip is given by $(-4\pi\rho\zeta e^{i\pi/6} - 2)$ from Eq. (17) together with $\rho_1 = \rho$. The amplitude gain and phase shift amount to 0.5 and 1.4, respectively, for $-\rho\zeta = 0.23$ or $2(-4\pi\rho\zeta)^3 = 50$. In the case of an oscillator with higher gain, the gain and phase shift become grater than the above values. It is therefore unclear whether the above gain and phase shift are consistent with the assumption used for derivation of Eq. (12) that the field remains unchanged during the evolution. The consistency will be confirmed, if Eq. (19) agrees well with ρ_n obtained from a time dependent calculation using Eqs. (3), (4), (5) and (6). The calculation for an input field given by Eq. (9) with $\rho = 0.0044$ is performed under an assumption that an optical cavity loss is 0 and the amplitude $|a(0)|$at $\zeta = 0$ is constant. The parameter ρ_n at ζ is obtained from the slope of the amplitude or phase of the calculated field at the round-trip number n. The ratios of ρ_n to ρ obtained from those slopes at $\zeta = -0.23$ with respect to n are shown in Fig. 2. They agree well with Eq. (19). The calculations with input fields with different ρs show also similar results.

CONCLUSION

The field on the leading edge at the 1st round trip is identical with SASE and is given by the exponentially increasing term of the solution of the cubic equation. The FEL interaction between the field similar to SASE and electrons can be analyzed under an assumption that the field remains almost unchanged during the evolution. The analysis shows that the field on the leading edge scales with the FEL parameter ρ and the round-trip number n when the gain is much smaller than an optical cavity loss. The validity of the assumption that the field remains almost unchanged during pass through an undulator is also confirmed.

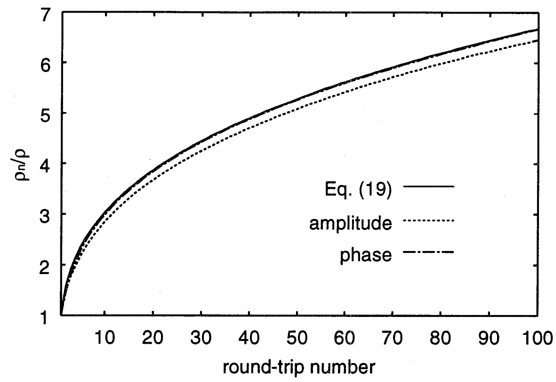

Figure 2: The ratio of ρ_n to ρ as a function of the round-trip number n. The solid line shows Eq. (19). The dashed line and dash-dotted line are derived from the slopes of the amplitude and phase of the calculated field, respectively.

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